ALGEBRA 1:

Week of April 13

Go through the slides (notes) and work through the examples on a separate piece of paper. Then do the given practice problems (again, on a separate piece of paper). Check your answers with the key given at the bottom of the practice page.

Check Google Classroom for the schedule of online help sessions via Zoom.

ALGEBRA: Lesson 3.5

Converting $x^2 + bx + c$ to Factored Form

Convert quadratic expressions in the form $x^2 + bx + c$ to factored form.

Explore! Number Riddles (This was the last HW assigned – see parentheses for solutions.)

Step 1 I am thinking of two integers that have a sum of 11 and a product of 24. What are my two numbers? **(8 & 3)**

Step 2 I am thinking of two integers that have a sum of 2 and a product of −35. What are my two numbers? **(7 & -5)**

Step 3 Are there only two integers that work for each of the above riddles? Explain your reasoning. (yes, otherwise you only get the sum OR the product, not both)

Step 4 Find two integers that add to the first number and multiply to the second number.

a.	9 and 20 (4&5)	**b.**	12 and 32 (4&8)	**c.**	7 and 12 (3&4)
d.	3 and −18 (6&-3)	**e.**	−5 and −14 (-7&2)	**f.**	7 and −30 (10&-3)
g.	4 and −45 (9&-5)	**h.**	0 and −16 (4&-4)	**i.**	−12 and 20 (-10&-2)

Step 5 Write two number riddles of your own that fit the description in **Step 4**. Have a classmate solve your riddles.

Step 6 How might a multiplication table be helpful in solving this type of riddle?

Factoring $x^2 + bx + c$

A quadratic expression in the form $x^2 + bx + c$ can be written in the form $(x + p)(x + q)$ if $p + q = b$ and $pq = c$.

$$\mathbf{x^2 + bx + c = (x + p)(x + q)}$$

F O I L

$$\begin{aligned} (x + 5)(x + 3) &= x^2 + 3x + 5x + 15 \\ &= x^2 + (3 + 5)x + 15 \\ &= x^2 + 8x + 15 \end{aligned}$$

The b value (OI) is the **sum** of the two numbers, 3 and 5.

The c value (L) is the **product** of the two numbers, 3 and 5.

Example 1

Factor $x^2 + 9x + 18$.

Find the values of b and c. $b = 9$ and $c = 18$

Make a list of factor pairs of c. Look for factors that have a sum equal to the value of b.

Factor Pairs of 18	Sum
1 and 18	19
2 and 9	11
3 and 6	9

The product of 3 and 6 equals 18 (the value of c). The sum of 3 and 6 equals 9 (the value of b).

Write in factored form. $x^2 + 9x + 18 = (x + 3)(x + 6)$

The Commutative Property says you can multiply the factors in either order: $(x + 3)(x + 6)$ or $(x + 6)(x + 3)$.

Check by distributing. ☑ $(x + 3)(x + 6) = x^2 + 6x + 3x + 18$
$= x^2 + 9x + 18$

Example 2

Factor each trinomial.

a. $x^2 - 4x - 32$

Find the values of b and c. $b = -4$ and $c = -32$

Make a list of factor pairs of c. Look for factors that have a sum equal to the value of b.

Factor Pairs of −32	Sum
1 and −32	−31
−1 and 32	31
2 and −16	−14
−2 and 16	14
4 and −8	−4
−4 and 8	4

The product of 4 and −8 equals −32 (the value of c). The sum of 4 and −8 equals −4 (the value of b).

Example 2 Continued…

Factor each trinomial.

a. $x^2 - 4x - 32$

Factor Pairs of −32	Sum
4 and −8	−4

Write in factored form. $x^2 - 4x - 32 = (x + 4)(x - 8)$

Check by distributing. ☑$(x + 4)(x - 8) = x^2 - 8x + 4x - 32$
$= x^2 - 4x - 32$

Example 2 Continued…

Factor each trinomial.

b. $x^2 - 10x + 21$

Find the values of *b* and *c*. $b = -10$ and $c = 21$

Make a list of factor pairs of *c*. Notice that the product is positive and the sum is negative. This means both numbers must be negative.

Factor Pairs of 21	Sum
−1 and −21	−22
−3 and −7	−10

$x^2 - 10x + 21 = (x - 3)(x - 7)$

Check by distributing. ☑$(x - 3)(x - 7) = x^2 - 7x - 3x + 21$
$= x^2 - 10x + 21$

Example 3

Graph the quadratic function $y = x^2 + 2x - 8$. Clearly mark the *x*-intercepts and the vertex.

Find the values of *b* and *c*. $b = 2$ and $c = -8$

Make a list of factor pairs of *c*. Look for factors that have a sum equal to the value of *b*.

Factor Pairs of −8	Sum
1 and −8	−7
−1 and 8	7
2 and −4	−2
−2 and 4	2

Write in factored form. $y = (x - 2)(x + 4)$

Example 3 Continued…

Graph the quadratic function $y = x^2 + 2x - 8$. Clearly mark the *x*-intercepts and the vertex.

Find the *x*-intercepts using the Zero Product Property.

Set the equation equal to zero. $0 = (x - 2)(x + 4)$

Set each factor equal to zero.

$$\begin{array}{rcl} 0 &=& x - 2 \\ +2 & & +2 \\ \hline 2 &=& x \end{array} \qquad \begin{array}{rcl} 0 &=& x + 4 \\ -4 & & -4 \\ \hline -4 &=& x \end{array}$$

Solve each equation for *x*.

The *x*-intercepts of the function are (2, 0) and (–4, 0).

Example 3 Continued…

Graph the quadratic function $y = x^2 + 2x - 8$. Clearly mark the *x*-intercepts and the vertex.

Find the axis of symmetry by averaging the two *x*-intercepts or using the formula $x = -\frac{b}{2a}$.

$$x = \frac{2 + (-4)}{2} = -1$$

Substitute $x = -1$ into the original function.

$$y = (-1)^2 + 2(-1) - 8$$
$$y = 1 + (-2) - 8$$
$$y = -9$$

Evaluate.

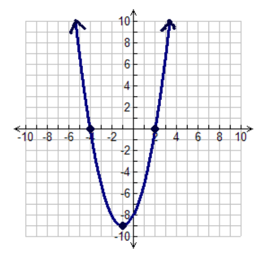
The vertex is at (−1, −9).

Example 3 Continued…

Graph the quadratic function $y = x^2 + 2x - 8$. Clearly mark the *x*-intercepts and the vertex.

Find two more points – one to each side of the vertex.

x = -2	x = 0
$y = (-2)^2 + 2(-2) - 8$	$y = 0^2 + 2(0) - 8$
$= 4 - 4 - 8$	$= -8$
$= -8$	
(-2, -8)	(0, -8)



Graph the five points (*x*-intercepts, vertex, and the two other points) and connect with a smooth curve.

3.5 Practice Problems: Factor each quadratic expression

1. $x^2 + 12x + 20$
2. $x^2 + 10x + 9$
3. $x^2 – 3x – 10$
4. $x^2 + 2x – 24$
5. $x^2 + 8x + 16$
6. $x^2 – 9x + 14$
7. $x^2 – 4x – 12$
8. $x^2 + 13x + 22$

3.5 Practice Problems: Find the zeroes of each quadratic function

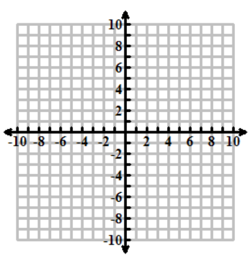
9. $y = x^2 + 10x + 21$

10. $p(x) = x^2 – 11x – 26$

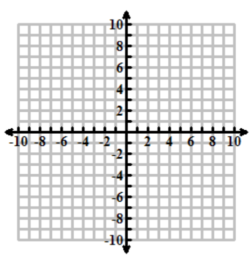
11. $g(x) = x^2 + x – 12$

3.5 Practice Problems: Graph each quadratic function. Clearly mark the *x*-intercepts and the vertex plus 2 more points. (Hint: Factor first, then find the zeroes).

$h(x) = x^2 + 2x - 8$



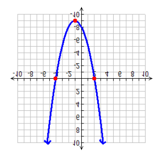
$y = x^2 + 6x + 5$



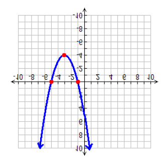
3.5 Practice Problems: ANSWER PAGE

- 1. $(x + 10)(x + 2)$
- 2. $(x + 9)(x + 1)$
- 3. $(x - 5)(x + 2)$
- 4. $(x + 6)(x - 4)$
- 5. $(x + 4)(x + 4)$ or $(x + 4)^2$
- 6. $(x - 7)(x - 2)$
- 7. $(x - 6)(x + 2)$
- 8. $(x + 11)(x + 2)$
- 9. $(x - 20)(x - 5)$
- 10. $x = -7$ and $x = -3$
- 11. $x = 13$ and $x = -2$
- 12. $x = -4$ and $x = 3$

Zeroes: x = -4 and x = 2
Vertex: (-1, -9)



Zeroes: x = -5 and x = -1
Vertex: (-3, -4)



ANSWERS TO LAST WEEK'S PACKET

Problems of the Day:

1) 91 pennies
2) 89 ways
3) 1)8 2)12 3)6 4)1
   4x4x4: 8, 24, 24, 8 5x5x5: 8, 36, 54, 27
4) 6, 6, 0
5) 15, 55, ½(n^2 + n)

ANSWERS TO LAST WEEK'S PACKET

Problems of the Day:

6) 5, 13, 26, 45 (+8, +13, +19)
7) 1st digit is 1 less than subtracting the first digits. 1st & last digit add to 9; middle digit =9
8) 6, 24, 120. use a factorial: "!" (! means if there are 6 blocks, it is 6! = 6*5*4*3*2*1)
9) They both needed 6 helpers
10) A square of 10x10 has the largest area.

ALGEBRA: Lesson 3.6

Converting $ax^2 + bx + c$ to Factored Form

Convert quadratic expressions in the form $ax^2 + bx + c$ to factored form.

Good to Know!

In the last lesson, you worked with quadratic expressions that had an a value equal to 1.

$$x^2 + bx + c = \underline{1}x^2 + bx + c.$$

You can factor $ax^2 + bx + c$ expressions by examining the **first** and **last** term in the expression. These terms give you clues about the two factors of the polynomial.

The last terms must multiply to 5.

$$3x^2 + 16x + 5 \rightarrow (\boxed{}\,x + \boxed{})(\boxed{}\,x + \boxed{})$$

The first terms must multiply to $3x$.

Terms that multiply to $3x^2$: $1x$ and $3x$

Terms that multiply to 5: 1 and 5

Good to Know!

$$3x^2 + 16x + 5 \rightarrow (\boxed{}\,x + \boxed{})(\boxed{}\,x + \boxed{})$$

At this point, use trial and error to try the different combinations in the empty slots above. Check if you have found the correct solutions. Find the product of the two "outside" terms. Add this to the product of the two "inside" terms.

If the two products sum to the middle term of the original quadratic expression, you have found the correct factors of the expression.

Trial 1: $(x + 1)(3x + 5)$

$$\begin{array}{r} 3x \\ +\ 5x \\ \hline 8x \end{array}$$

$\boxed{8x \neq 16x}$

Trial 2: $(x + 5)(3x + 1)$

$$\begin{array}{r} 15x \\ +\ x \\ \hline 16x \end{array}$$

$\boxed{16x = 16x}$

$$3x^2 + 16x + 5 \rightarrow (x + 5)(3x + 1)$$

Example 1

Factor $2x^2 + 7x + 6$.

Find the factor pairs of $2x^2$. $2x$ and x

Find the factor pairs of 6. 1 and 6 <u>or</u> 2 and 3

Check each possible combination until you find the right one.

$(2x + 1)(x + 6)$

$2x^2 + 13x + 6$
Incorrect

$(2x + 6)(x + 1)$ $(2x + 2)(x + 3)$

These factors have a common factor inside of the parentheses because 2 can be divided into both terms. The original expression did not have a common factor throughout, so these cannot be the solution.

$(2x + 3)(x + 2)$

$2x^2 + 7x + 6$
Correct!

$$\mathbf{2x^2 + 7x + 6 = (2x + 3)(x + 2)}$$

Factoring $ax^2 + bx + c$

1. Find pairs of factors that multiply to the first term, ax^2.
2. Find pairs of factors that multiply to the last term, c.
3. Create possible factored sets from the combinations in Steps 1 and 2. Multiply each binomial to see if it expands to equal the original expression.

Good to Know!
When trinomials involve negative numbers, you will find that there are more possibilities to try. Look for patterns with the negative factors that may lead to the correct expression when multiplied.

Example 2

Factor $3x^2 - 11x - 4$.

Find the factor pairs of $3x^2$. $3x$ and x

Find the factor pairs of -4.
1 and -4
<u>**or**</u> -1 and 4
<u>**or**</u> 2 and -2

Check each possible combination until you find the right one.

$(3x + 1)(x - 4)$
$3x^2 - 11x - 4$
Correct!

$(3x - 1)(x + 4)$ $(3x + 2)(x - 2)$ $(3x - 2)(x + 2)$

No need to expand these once you have found a combination that works.

$$\mathbf{3x^2 - 11x - 4 = (3x + 1)(x - 4)}$$

Example 3

Factor $4x^2 - 8x = -3$.

Move term(s) so that the equation is in the form $ax^2 + bx + c = 0$.

$$\begin{aligned} 4x^2 - 8x \quad &= -3 \\ +3 \quad &\;\; +3 \\ \hline 4x^2 - 8x + 3 &= 0 \end{aligned}$$

Find the factor pairs for $4x^2$. $4x$ and x <u>or</u> $2x$ and $2x$

Find the factor pairs for 3. 1 and 3 <u>or</u> -1 and -3

Example 3 Continued…

Factor $4x^2 - 8x = -3$.

Check each possible combination until you find the right one.

$(2x + 1)(2x + 3)$	$(2x - 1)(2x - 3)$	$(x + 1)(4x + 3)$	$(x - 1)(4x - 3)$
The middle term is negative so this expression will not work.	$4x^2 - 6x - 2x + 3$ $4x^2 - 8x + 3$ **YES!**	The middle term is negative so this expression will not work.	**Not Correct**

Since $4x^2 - 8x + 3 = (2x - 1)(2x - 3)$, set each factor each to 0 and solve.

$$\begin{aligned} 2x - 1 &= 0 \quad \text{and} \quad 2x - 3 = 0 \\ +1 \;\; &+1 \qquad\qquad\;\; +3 \;\; +3 \end{aligned}$$

$$x = \frac{1}{2} \quad \text{and} \quad x = \frac{3}{2}$$

The solutions are where the graph of the function $f(x) = 4x^2 - 8x + 3$ crosses the x-axis.

<u>3.6 Practice Problems</u>: Factor each quadratic expression

You only need to list the positive combinations since all terms are positive.

1. $2x^2 + 11x + 5$

a. List the factor combination that multiply to $2x^2$: _______ and _______

b. List the factor combination that multiplies to 5: _______ and _______

c. Try each possibility to see which one equals $2x^2 + 11x + 5$.

(____ + ____)(____ + ____) (____ + ____)(____ + ____)

<u>3.6 Practice Problems</u>: Factor each quadratic expression

2. $5x^2 + 17x + 6$

3. $3x^2 + 5x + 2$

4. $3x^2 + 5x - 2$

5. $4x^2 + 8x + 3$

6. $2x^2 - 17x + 21$

7. $3x^2 + 4x - 4$

<u>3.6 Practice Problems</u>: Find the zeros of each quadratic function. (Hint: Set each function equal to zero. Factor and then solve.)

8. $f(x) = 2x^2 + 9x + 7$

9. $h(x) = 5x^2 + 4x - 1$

Solve each equation. (Hint: see example 3.)

10. $3x^2 + 17x = -10$

11. $2x^2 - x = 15$

3.6 Practice Problems: ANSWER PAGE

- 1.
  - a. $2x$ and x
  - b. 5 and 1
  - c. $(2x + 1)(x + 3)$
- 2. $(5x + 2)(x + 3)$
- 3. $(3x + 2)(x + 1)$
- 4. $(3x - 1)(x + 2))$
- 5. $(2x + 3)(2x + 1)$
- 6. $(2x - 3)(x - 7)$
- 7. $(3x - 2)(x + 2)$

8. x = -7/2 and x = -1

9. x = 1/5 and x = -1

10. x = -2/3 and x = -5

11. x = -5/2 and x = 3

ALGEBRA: LAST SLIDE for this week!