ALGEBRA 1: Week of May 4

- Go through the slides (notes) and work through the examples on a separate piece of paper.
- Do the given practice problems (again, on a separate piece of paper).
- Check your answers with the key given (last slide).
- Take a photo or scan in your work and submit it in Google Classroom. If you have questions or would like feedback on your work, add that as a comment with your submitted work.
- The other option for turn in is to send it in on Monday when the new packet is available.
- Check your school email/google calendar for online help sessions via Zoom.

Day 1: Lesson 3.8

The Quadratic Formula

Target: Use the Quadratic Formula to solve quadratic equations.

The Quadratic Formula is a method which can be used to solve quadratic equations in the form $\mathbf{0 = ax^2 + bx + c}$, where $\mathbf{a \neq 0}$.

The zeros are given by the formula:

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

Example 1

Find the solutions to the quadratic equation: $0 = 2x^2 + 3x - 20$.

List the *a*, *b* and *c* values. $a = 2,\ b = 3,\ c = -20$

Write parentheses in place of each variable in the Quadratic Formula.

$$x = \frac{-(\ ) \pm \sqrt{(\ )^2 - 4(\ )(\ )}}{2(\ )}$$

Substitute the values of *a*, *b* and *c* into the correct locations.

$$x = \frac{-(3) \pm \sqrt{(3)^2 - 4(2)(-20)}}{2(2)}$$

Example 1 Continued…

Find the solutions to the quadratic equation: $0 = 2x^2 + 3x - 20$.

Simplify by following the order of operations.

$$x = \frac{-3 \pm \sqrt{9 - (-160)}}{2(2)}$$

Subtract under the grouping symbol.

$$x = \frac{-3 \pm \sqrt{169}}{2(2)}$$

Simplify $\sqrt{169}$ and multiply the denominator.

$$x = \frac{-3 \pm 13}{4}$$

Remember when you square root you get a positive and a negative.

Example 1 Continued…

Find the solutions to the quadratic equation: $0 = 2x^2 + 3x - 20$.

Find the two solutions by using both the + and −.
(be sure to either push "=" before dividing or to put the numerator in parentheses)

$$x = \frac{-3 + 13}{4} = 2.5$$

$$x = \frac{-3 - 13}{4} = -4$$

The solutions to the quadratic equation are $x = -4$ and $x = 2.5$.

Good to Know!

Not all solutions to quadratic equations will result in rational answers (numbers that can be written as fractions using integers).

The Quadratic Formula helps you find solutions to quadratic equations that have irrational answers. You can leave these as exact answers (with the square root) or round the solutions to give approximate answers.

Example 2

Solve the equation $3x^2 - 7x = 5$. Write the solutions as exact values and approximations rounded to the hundredth.

Rewrite in general form by moving the constant. $3x^2 - 7x - 5 = 0$

List the *a*, *b* and *c* values. $a = 3,\ b = -7,\ c = -5$

Substitute the values of *a*, *b* and *c* into the correct locations.

$$x = \frac{-(-7) \pm \sqrt{(-7)^2 - 4(3)(-5)}}{2(3)}$$

$-(-7)$ makes the 7 positive

Example 2 Continued…

Solve the equation $3x^2 - 7x = 5$. Write the solutions as exact values and approximations rounded to the hundredth.

Evaluate exponents and multiplication under the grouping symbol. $x = \dfrac{7 \pm \sqrt{49 - (-60)}}{2(3)}$

Subtract under the grouping symbol and multiply the denominator. $x = \dfrac{7 \pm \sqrt{109}}{6}$

The exact solutions to the quadratic equation are:

$$x = \frac{7 + \sqrt{109}}{6} \quad \text{and} \quad x = \frac{7 - \sqrt{109}}{6}$$

The approximate solutions are $x \approx 2.91$ and $x \approx -0.57$

Day 1 Practice Problems

Rewrite each quadratic equation in general form. Identify the values of *a*, *b* and *c*.

1. $x^2 - 6 = 11x$
2. $4x^2 = x + 7$
3. $12 - 5x = -2x^2$

Solve each quadratic equation using the Quadratic Formula. Round answers to the nearest tenth, if necessary.

4. $0 = x^2 + 11x + 30$
5. $0 = 2x^2 - x - 3$
6. $0 = 3x^2 + 19x + 6$
7. $x^2 + 7 = 6x$
8. $2x^2 = -9x + 12$
9. $4x^2 + x - 4 = 0$

10. Xavier threw a ball off of a cliff. The ball's height as a function of time could be modeled by the following function, where *t* is the time in seconds and *h* is the height in feet. After how many seconds will the ball land on the ground?

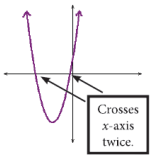
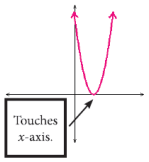
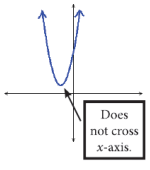
$h(t) = -16t^2 + 16t + 92$

End Day 1

Day 2

Good to Know!

The expression under the radical sign, $b^2 - 4ac$, in the Quadratic Formula is called the ***discriminant***. The discriminant gives you information about the number of real roots or solutions of a quadratic equation. Some quadratics have two *x*-intercepts, some have one and others do not cross the *x*-axis at all.

Discriminant	$b^2 - 4ac > 0$ POSITIVE	$b^2 - 4ac = 0$ ZERO	$b^2 - 4ac < 0$ NEGATIVE
Graph			
Number of Real Roots or Solutions	2	1	0

Example 3

Determine the number of real solutions of each quadratic equation using the discriminant.

a. $x^2 - 8x + 16 = 0$

List the *a*, *b* and *c* values. $a = 1,\ b = -8,\ c = 16$

Substitute the values of *a*, *b* and *c* into $b^2 - 4ac$. $(-8)^2 - 4(1)(16)$

Evaluate. $64 - 64 = 0$

The discriminant equals 0, so the quadratic equation has ONE real solution.

Example 3 Continued…

Determine the number of real solutions of each quadratic equation using the discriminant.

b. $2x^2 - 5x + 9 = 0$

List the *a*, *b* and *c* values. $a = 2,\ b = -5,\ c = 9$

Substitute the values of *a*, *b* and *c* into $b^2 - 4ac$. $(-5)^2 - 4(2)(9)$

Evaluate. $25 - 72 = -47$

The discriminant is negative, so the quadratic equation has NO real solutions.

Example 3 Continued…

Determine the number of real solutions of each quadratic equation using the discriminant.

c. $4x^2 = -3x + 10$

Rewrite the equation in general form. $4x^2 + 3x - 10 = 0$

List the *a*, *b* and *c* values. $a = 4,\ b = 3,\ c = -10$

Substitute the values of *a*, *b* and *c* into $b^2 - 4ac$. $(3)^2 - 4(4)(-10)$

Evaluate. $9 - (-160) = 169$

The discriminant is positive, so the quadratic equation has TWO real solutions.

Day 2 Practice Problems

Determine the value of the discriminant. State the number of real solutions.

11. $3x^2 - 10x + 8 = 0$ 12. $16x^2 = -24x - 9$ 13. $2x^2 + 9 = -7x$

14. Victor found the value of the discriminant of a quadratic equation and said the graph of the quadratic function would have no real *x*-intercepts. What does this tell you about the value of the discriminant?

$$f(x) = 4x^2 - 25$$

15. Petra determined that the graph of the following quadratic function has two *x*-intercepts. Do you agree with her? Why or why not?

End Day 2

Day 3 Practice Problems: Applications

Use the vertical motion model for problems 1-3: $h = -16t^2 + vt + s$

h = height (feet), *t*= time (seconds) *v* = velocity, *s* = initial height (feet)

1. An acrobat is shot out of a cannon and lands in a safety net that is 10 feet above the ground. Before being shot out of the cannon, she was 4 feet above the ground. She left the cannon with an initial upward velocity of 50 feet per second. Find the time t (in seconds) it takes for her to reach the net.

A. Write the equation, substituting the information given above: *h* = _____

B. What will *h* be when she lands in the net? ____. New equation: _____

C. To solve, your equation must = 0. Add or subtract to make it this way, then solve.

D. Explain why only one of the two solutions is reasonable.

Practice Problems: Applications

2. At a basketball game, T-shirts are rolled-up into a ball and shot from a "T-shirt cannon" into the crowd. The T-shirts are released from a height of 6 feet with an initial upward velocity of 44 feet per second. If you catch a T-shirt at your seat 30 feet above the court, how long was it in the air before you caught it?

A. Write the equation, substituting the information given above: *h* =_____

B. What will *h* be when you catch the T-shirt? ____. New equation: _____

C. To solve, your equation must = 0. Add or subtract to make it this way, then solve.

3. An object is propelled from the ground with an initial upward velocity of 224 feet per second. Will the object reach a height of 784 feet? If it does, how long will it take the object to reach that height?

A. Write the equation, substituting the information given above: *h* = _____

B. What is the *h* you're looking for? _____. New equation: _____

C. To solve, your equation must = 0. Add or subtract to make it this way, then solve by factoring:

Practice Problems: Applications

Use the hang time model: $h = 4t^2$ *h* = height jumped (feet), *t* = hang time (seconds)

4. When you jump, your "hang time" is the amount of time that passes while you are in the air.

a. If you jump 1 foot into the air, what is your hang time?

b. If a professional basketball player jumps 4 feet into the air, what is the hang time?

Write and Solve a quadratic equation for problems 5-6:

5. A rectangular frame has an area of 72 square inches. The length is represented by $x + 2$ inches. The width is represented by the expression $2x - 3$ inches. What are the lengths of the sides? [*Hint: start by substituting the given information into the rectangle area formula: $A = lw$ and then use FOIL]

6. Mr. Smith said the length and width of his rectangular classroom could be represented by the expressions $3x - 5$ and $4x - 3$. The area of the room is 273 square feet. What are the dimensions of the classroom?

End Day 3

ANSWER PAGE

Day 1:

1. $0 = x^2 - 11x - 6$; $a = 1, b = -11, c = -6$
2. $0 = 4x^2 - x - 7$; $a = 4, b = -1, c = -7$
3. $0 = -2x^2 + 5x - 12$; $a = -2, b = 5, c = -12$
4. $x = -5$ and $x = -6$
5. $x = 1.5$ and $x = -1$
6. $x \approx -0.3$ and $x = -6$
7. $x \approx 4.4$ and $x \approx 1.6$
8. $x \approx 1.1$ and $x \approx 5.6$
9. $x \approx 0.9$ and $x \approx -1.1$
10. About 2.9 seconds

Day 2:

11. 4; 2 real solutions
12. 0; 1 real solution
13. –23; 0 real solutions
14. The discriminant had a negative value.
15. Yes. The discriminant is positive.

Day 3:

1. a) $-16t^2+50t+4$ b) $10 = -16t^2+50t+4$
c) $t = \frac{1}{8}$ & 3 d) The correct answer choice is 3 seconds, because at 1/8 of a second, she would be flying upward into the net (ouch!).
2. a) $-16t^2+44t+6$ b) $30=-16t^2+44t+6$
c) 2 seconds ($\frac{3}{4}s$ would be on the way up)
3. a) $-16t^2+224t$ (ground = 0) b) $784=-16t^2+224t$
c) $0 = -16(t-7)^2$ $t = 7$s
4. a) $\frac{1}{2}s$ b) 1s
5. $x = 6$ & -6.5 ; dimensions: 9 in x 8 in
6. $x = 3$ & $-3\frac{7}{12}$; dimensions: 13 ft x 21 ft

ALGEBRA: LAST SLIDE for this week!